QUIZ 21 SOLUTIONS: LESSON 28
NOVEMBER 10, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] If R is the region bounded by the x-axis, $y = \cos(x)$, $x = 0$, and $x = \dfrac{\pi}{4}$, find

$$\iint_R \sec^3(x)\, dA.$$

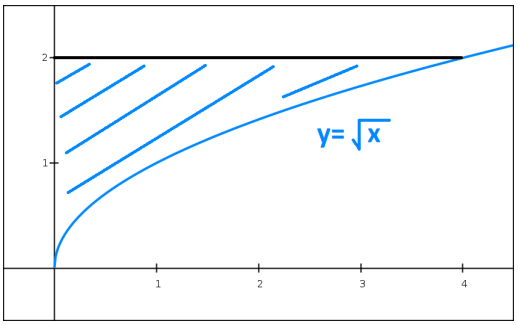
Solution: We are given $0 \leq x \leq \dfrac{\pi}{4}$ and $0 \leq y \leq \cos(x)$. Hence

$$\begin{aligned}
\iint_R \sec^3(x)\, dA &= \int_0^{\pi/4} \int_0^{\cos(x)} \sec^3(x)\, dy\, dx \\
&= \int_0^{\pi/4} y \sec^3(x) \Big|_{y=0}^{y=\cos(x)} dx \\
&= \int_0^{\pi/4} \cos(x) \sec^3(x)\, dx \\
&= \int_0^{\pi/4} \sec^2(x)\, dx \\
&= \tan(x) \Big|_{x=0}^{x=\pi/4} \\
&= \tan\left(\frac{\pi}{4}\right) - \tan(0) \\
&= 1 - 0 = \boxed{1}
\end{aligned}$$

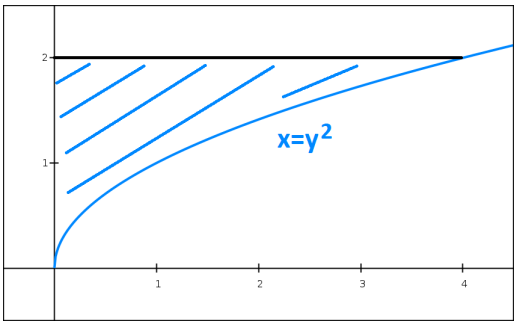
2. [5 pts] Evaluate

$$\int_0^4 \int_{\sqrt{x}}^2 9(y^3+1)^{1/2}\,dy\,dx.$$

Solution: We need to swap the order of integration. Our region is described by $0 \le x \le 4$ and $\sqrt{x} \le y \le 2$. Sketching a picture, we get



Now, putting things in terms of x, we can describe the region by $0 \le y \le 2$ and $0 \le x \le y^2$.



Therefore,

$$\begin{aligned}\int_0^4 \int_{\sqrt{x}}^2 9(y^3+1)^{1/2}\,dy\,dx &= \int_0^2 \int_0^{y^2} 9(y^3+1)^{1/2}\,dx\,dy \\ &= \int_0^2 9x(y^3+1)^{1/2}\Big|_{x=0}^{x=y^2}\,dy \\ &= \int_0^2 9y^2(y^3+1)^{1/2}\,dy\end{aligned}$$

This is now a u-substitution problem. Let $u = y^3+1$, then $du = 3y^2\,dy$, $u(0) = 0^3+1 = 1$, and $u(2) = 2^3+1 = 9$. Hence,

$$\begin{aligned}\int_0^2 9y^2(y^3+1)^{1/2}\,dy &= \int_1^9 3u^{1/2}\,du \\ &= 2u^{3/2}\Big|_1^9\end{aligned}$$

$$= 2((9)^{3/2} - 1^{3/2})$$

$$= 2(3^3 - 1)$$

$$= 2(27 - 1)$$

$$= 2(26) = \boxed{52}$$